## Exponentials and logarithms 14A

1 a

b Where $y=4, x \approx 2.6$

2 a

b Where $y=2$, $x \approx-1.4$
3


4 a True because, when $x=0, a^{0}=1$ when $a$ is positive
b False. For example, when $a=\frac{1}{2}$, the function $\mathrm{f}(x)=a^{x}$ is not an increasing function.
c True because, when $a$ is positive, $\mathrm{a}^{x}>0$ for all values of $x$.

5 a The graph crosses the $y$-axis when $x=0$.
$y=3^{0}$
So $y=1$
The graph crosses the $y$-axis at $(0,1)$. Asymptote is at $y=0$.

b The graph is a vertical stretch by scale factor 2.
The graph crosses the $y$-axis when $x=0$.
$y=2 \times 3^{0}$
So $y=2$
The graph crosses the $y$-axis at $(0,2)$.
Asymptote is at $y=0$.

c The graph is a translation by the vector $\binom{0}{-4}$.
The graph crosses the $y$-axis when $x=0$.
$y=3^{0}-4$
So $y=-3$
The graph crosses the $y$-axis at $(0,-3)$.
Asymptote is at $y=-4$.

5 c

d The graph is a horizontal stretch by scale factor 2.
The graph crosses the $y$-axis when $x=0$.
$y=3^{\frac{1}{2} \times 0}$
So $y=1$
The graph crosses the $y$-axis at $(0,1)$.
Asymptote is at $y=0$.


6 Substitute the coordinates into $y=k a^{x}$.
$6=k a^{1}$ (equation 1 )
$48=k a^{4}$ (equation 2)
Solve simultaneously: divide equation 2 by equation 1 ,

$$
\begin{aligned}
& 48 \div 6=\frac{k a^{4}}{k a} \\
& a^{3}=8 \\
& a=2, k=3
\end{aligned}
$$

7 a As $x$ increases, $y$ decreases

b Substitute the coordinates into $y=p q^{x}$. $150=p q^{-3}$ (equation 1 )
$0.048=p q^{2}$ (equation 2)
Solve simultaneously, divide equation 2 by equation 1.
$0.048 \div 150=\frac{p q^{2}}{p q^{-3}}$
$q^{5}=0.00032$
$q=0.2$
$p=0.048 \div 0.2^{2}=1.2$
$p=1.2, q=0.2$

## Challenge

To draw the graph, note that it is a translation of the graph $y=2^{x}$ by the vector $\binom{2}{5}$.
The graph crosses the $y$-axis when $x=0$, so $y=2^{0-2}+5$ $y=5.25$
The graph crosses the $y$-axis at $(0,5.25)$. Asymptote is at $y=5$.


